Learning the parameters of a multidimensional spatial preference model in multi-criteria decision aiding

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Abstract. In this work we propose a new way to represent preferences of multiple decision makers in multi-criteria decision aiding, through a multidimensional spatial model. The decision makers are represented in a shared space with the alternatives so that their positions are consistent with the preferences that they express on pairs of alternatives. We propose different mathematical program formulations in the case of multiple decision makers (independent or dependent DMs) to learn the parameters of this preference model.

1 Introduction and related work

In the spatial theory of voting, voters and candidates are represented as points in a space defined by a set of attributes or political issues. The position of the voters and the candidates is defined by the way in which they espouse these issues. The main purpose of the spatial voting model is to describe and analyse the voters' behaviours and to estimate the outcome of an election using the distance between the voters and the candidates. The objective of multi-criteria decision aiding is to provide a recommendation to a decision maker, confronted with a set of alternatives, described through a set of potentially conflicting criteria, by taking into account his or her preferences. In many real-world decision problems it may be interesting to understand the behavior of the decision maker when faced with these alternatives or to compare the behavior of several decision makers when faced with these alternatives. These observations lead us to draw a parallel with what is done in the spatial voting theory. The decision maker could be represented in the criteria space so that his or her position is consistent with the preferences that he or she expresses on pairs of alternatives. The idea is as follows: if he or she says that he or she prefers alternative a to alternative b, then the point representing the decision maker in the criteria space must be closer to a than to b. Similarly, if he or she also says that he or she prefers c to d, then the point representing the decision maker must also be closer to c than to d, while being closer to a than to b.

Positioning the decision maker in the same multidimensional space as the alternatives can be used for different purposes. Similarly as in the spatial theory of voting, this paradigm allows us to explain to a decision maker in which part of the criteria space he or she is located, and consequently, which alternatives can be the best for him or her. Moreover, it allows us to position different decision makers in this space in order to evaluate their proximity, or to determine on which alternatives their preferences are similar or dissimilar. And last but not least, the preferences on the criteria do not have to be monotonically increasing or decreasing, as usually supposed in multi-criteria decision aiding.

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In multi-criteria decision aiding, different contributions propose to handle the case of non-monotonic preferences on criteria. In Multi-Attribute Value Theory (MAVT) the works of [2, 8, 9, 5, 7, 6] tackle this problem in various ways. For the outranking approach various preference elicitation techniques are proposed by [10] in such a a non-monotonic setting.

In this work, we introduce first the model in Section 2, before showing how the parameters of the model can be learnt through a holistic elicitation procedure in Section 3.

2 A multidimensional spatial model for preference representation in multi-criteria decision aiding

Let us consider a set of alternatives denoted $\mathcal{A}$. Each alternative $a \in \mathcal{A}$ is evaluated on a set of m attributes denoted $\mathcal{M} = \{1, \ldots, m\}$. Let a_j be the quantitative evaluation of a on attribute $j \in \mathcal{M}$, with $a_j \in \mathbb{R}$. a can be identified with its performance vector, i.e., $a \equiv (a_1, \ldots, a_m)$. As a consequence, an alternative $a \in \mathcal{A}$ can be represented by a point in a multidimensional space $\mathbf{E} = \mathbb{R}^m$, in which each dimension corresponds to one attribute of $\mathcal{M}$. We do not suppose in this work any preference direction given by the decision maker on the attribute scales.

The proposed decision model is based on the relative positions of the decision maker and the alternatives in the same multidimensional space.Let $\mathcal{X} = \{1, \ldots, n\}$ be the set of n decision makers. Let $\succ_i$ ($resp.$ $\sim_i$) $\subseteq \mathcal{A} \times \mathcal{A}$ be the strict preference (resp. the indifference) relation of ith decision maker $i \in \mathcal{X}$ on the alternatives of $\mathcal{A}$.

We define the spatial preference model (SP-model) as follows:

Definition 1. *(Spatial preference model) Let $i \in \mathcal{X}$. A preference relation $\succsim_i$ on $\mathcal{A}$ follows a spatial preference model if and only if*

- *there exists a representation $(x_1^i, \ldots, x_m^i)$ of X^i in $\mathbf{E}$*
- *there exists a distance δ on $\mathbf{E} \times \mathbf{E}$*

such that $\forall (a, b) \in \mathcal{A} \times \mathcal{A}$:

$$\begin{cases} a \succ_i b \iff \delta(a, X^i) < \delta(b, X^i) \\ a \sim_i b \iff \delta(a, X^i) = \delta(b, X^i). \end{cases}$$

The spatial preference model defined in Definition 1 is a utility-based model, and it therefore models total pre-orders on the set of alternatives. The hypothesis of single-peakedness on each attribute is the second characteristic of spatial preference models. A complete characterization of preference relations that can be represented by a spatial model using a Minkowski metric as distance has been proposed in [3] in the framework of political issues. This characterization is based on both utility functions and single-peakedness property.

A large number of distances can be used to model spatial preferences. Keeping in mind the aim that the model should be easily understandable by the decision-maker. We propose in the following to focus on the Euclidean distance as it is more simple to represent for the decision maker. However, with a fixed distance and without any utility functions, the model could sometimes be under-parameterized. Therefore, in order to capture the importance given by different decision makers to the same attributes, we propose to use an extension of the Euclidean distance, which weighs the various attributes differently, and gives further flexibility to the model. The weighted Euclidean distance is defined as follows for an alternative $a \in \mathcal{A}$ and $i \in \mathcal{X}$:

$$\delta(a, X^i) = \sqrt{\sum_{j \in \mathcal{M}} w_j^i \cdot (x_j^i - a_j)^2}, \tag{1}$$

where $w_j^i \geq 0$ and $\sum_{j=1}^{m} w_j^i = 1$.

The advantage of this weighted Euclidean distance compared to the classical Euclidean distance, is that it to a great flexibility of the model, by allowing for differences on the attributes to be more or less important in the distance calculation for each decision maker. Using different weights if needed, We therefore keep the main interest of the SP-model, that is to capture in a single explainable model preferences that are different for different decision makers.

3 Parameters inference

Usually in the spatial voting models, the unknown parameters are both the positions of the candidates (the decision makers for us) and the voters (the alternatives). The methods used in this case, as shown by Armstrong *et al.* [1], are not suitable for our paradigm where the positions of the alternatives are known, and where mainly two types of parameters have to be determined: the positions of the decision makers in $\mathbf{E}$ and the weights used in the weighted Euclidean distance. As classically done in Multi-Criteria Decision Aiding, they could be elicited in a direct way [4], by questioning the decision makers. However, as one or both of these elements are not necessarily known by the decision makers, we propose to use an indirect elicitation approach to learn the parameters of the model through a holistic approach.

Let us now present how the preference parameters of the proposed model can be determined by using mathematical programming techniques. The unknowns to be determined are the values of the parameters of the proposed model, i.e.:

1. the positions of the decision makers in the multidimensional space $\mathbf{E}$, $X^i \equiv (x_1^i, x_2^i, \ldots, x_m^i), \forall i \in \mathcal{X}$,
2. the weights of the different attributes w_j^i, for $j \in \mathcal{M}$ used in the distance, $\forall i \in \mathcal{X}$ (see Equation 1).

We propose three different formulations. In the first one we consider that all the decision makers are independent of each other and thus do not share any preferential parameters (neither the weights nor their positions). This means that there is no connection between them, and each of them has an independent preference model. Next to that, we assume here that the preference statements expressed by these decision makers are all compatible with the proposed model. To determine the values of these preference parameters in this first case, we model the problem as a non-linear mathematical program.

The idea behind the second formulation is that if there is no feasible solution compatible with the decision maker i's preferences, then we try to position X^i in such a way as to minimize an error measure. Intuitively, for each strict preference and indifference statement, we allow the distance condition to be violated by a certain amount. The goal of the mathematical program is then to minimize the total error, throughout all the decision makers. In case all the preference and indifference statements of all the decision makers are compatible with the proposed model, then there are no such errors, and the program comes down to that of the initial mathematical program.

The third one is in a group decision maker context, when it could makes sense to create a link between decision makers and to determine a dependent preference model. The goal could be to create groups of similar decision makers. In the proposed model, this interdependence can concern either the parameters of the distance or the positions of the decision makers. To learn such a common model, we propose to impose that all the decision makers share common weights.

We also study experimentally the behavior of the proposed elicitation model when facing artificially generated random data, for a single decision maker. The goal is to answer the following questions:

- How do resolution times vary with the number of input preference statements?
- How does the model generalize when confronted with unseen data?
- What is the influence of noise in the input preference judgments on the model?

We show also the interest of the approach in such a real world problem in the context of the TIGA project [11].

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