Preference disaggregation involving time-series: a new approach based on multi-objective optimization

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Abstract. Preference disaggregation in MCDA infers information from examples of decisions provided by the decision maker. In this respect, only a few studies consider the criteria value evolution over time. In our research, we provide a preliminary investigation of preference disaggregation when the criteria time-series are relevant. Also, a multi-objective mathematical model is proposed to explore multiple compatible models. We validate our approach using real data to rank countries in terms of Human Development Index.

1 Introduction

Multiple Criteria Decision Aid (MCDA) aims to model relations between a set of alternatives $A = \{a_1, \dots, a_m\}$, evaluated according to a set of criteria, $C = \{c_1, \dots, c_n\}$. UTA and UTASTAR [3] are MCDA disaggregation methods used to infer marginal value functions from a given ranking on a set of alternatives. These methods lead to multiple compatible decision models which involve a robustness issue [2].

MCDA disaggregation methods have in common that each criterion is associated with a single value, which may be the average of the criterion's performance in a given period of time or some other static data. However, several decisions are made considering the evolution of the criterion over time, its time-series, or some characteristics of these time-series (summary measures such as the average and trend). For instance, suppose that to open a new country branch, the DM analyzes two criteria, gross domestic products and purchasing power parity. In this type of investment, the trend of these criteria is as relevant as their average; thus, for the model to be representative, it is important to infer the decision maker's (DM) preference considering both the average and trend.

Therefore, we propose UTASTAR-Tensors (or simply UTASTAR-T), an extension of UTASTAR, to infer the DM's preferences in a context where some characteristics of the time-series criteria are considered simultaneously. Besides, a multi-objective mathematical model is proposed to exploit the space of compatible decision models.

2 UTASTAR-Tensors

This section presents the UTASTAR-T algorithm and the multi-objective mathematical model proposed to generate multiple solutions. Our propose consists in inferring the DM's preferences when the characteristics of the criteria time-series are relevant in the decision.

In the UTA family methods the data are usually represented in a matricial formulation, in which the rows represent the alternatives and the columns the criteria. In this study, we depict the problem as a *tensor* (multidimensional arrays), where the third dimension corresponds to a time index, which means that a given criterion is represented by a time series. Then, we carry out a space transformation; the third dimension becomes the time-series summary measure, in this case, the criteria average and trend.

In this new approach, instead of estimating each criterion's utility function, $u_j(c_j)$, as in the classic UTA family methods, we infer the criterion's utility function for each characteristic k, denoted by $u_{kj}(c_{kj})$, $k = 1, \dots, h$. We have n criteria and h characteristics that compose $h \times n$ utility functions. The criterion evaluation scale of each criterion j for each summary measure k is denoted by $[c_{kj}^1, c_{kj}^2, \cdots, c_{kj}^{\alpha_{kj}}]$, where c_{kj}^1 is the worst value of criterion j in characteristic k and $c_{kj}^{\alpha_{kj}}$ the best one, and α_{kj} represents the number of different values criterion j assumes in each summary measure k.

The global value function of alternative i is represented by $u[\mathbf{c}(a_i)]$, $i = 1, \dots, m$, obtained as follows: $u[\mathbf{c}(a_i)] = \sum_{k=1}^{h} \sum_{j=1}^{n} u_{kj}(c_{kj})$, in which the following properties hold:

$$\begin{cases} u[\mathbf{c}(a_i)] > u[\mathbf{c}(a_{i+1})] \iff a_i \succ a_{i+1} \text{ (preference)} \\ u[\mathbf{c}(a_i)] = u[\mathbf{c}(a_{i+1})] \iff a_i \sim a_{i+1} \text{ (indifference)} \end{cases} \quad (1)$$

In the UTASTAR-T, first, the set of reference alternatives, $A_R = \{a_1, \dots, a_m\}$, is ordered in such a way that a_1 is the best and a_m the worst alternative according to the decision-maker ranking. Then, each pairwise comparison showed in (1) is translated into a constraint as follows: for each pair of alternatives $(a_i, a_{i+1}) \in A_R$, we have:

$$\Delta(a_i, a_{i+1}) = \big(u[\mathbf{c}(a_i)] - \sigma^+(a_i) + \sigma^-(a_i)\big) - \big(u[\mathbf{c}(a_{i+1})] - \sigma^+(a_{i+1}) + \sigma^-(a_{i+1})\big)\,, \quad (2)$$

in which $\sigma^-(a_i)$ represents the overestimation error, and $\sigma^+(a_i)$ the underestimation error.

The variables $w_{j\ell}$ used in the classical UTASTAR are replaced by $w_{kj\ell}$ in order to guarantee the monotonicity condition for all utility functions in each summary measure, in the form: $w_{kj\ell} = u_{kj}(c^{\ell+1}) - u_{kj}(c^{\ell}) \geq 0$, $\forall\, \ell = 1, \cdots, (\alpha_{kj} - 1), j = 1, \cdots, n, k = 1, \cdots, h$. Moreover, in UTASTAR-T, the variables $w_{kj\ell}$ are normalized in such a way that they sum up to 1: $\sum_{j=1}^{n} \sum_{\ell=1}^{\alpha_{kj}-1} w_{kj\ell} = 1$, for each k.

Therefore, the UTASTAR-T algorithm is as follows:

Step 1: Express the global value of each alternative as $u[\mathbf{c}(a_i)] = \sum_{k=1}^{h} \sum_{j=1}^{n} u_{kj}(c_{kj})$;
Step 2: Generate Constraints (2) for each $(a_i, a_{i+1}) \in A_R$;
Step 3: Express the variables $w_{kj\ell}$ as $w_{kj\ell} = u_{kj}(c^{\ell+1}) - u_{kj}(c^{\ell})$, $\forall j, \ell, k$;

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Step 4: Solve the linear program:

$$min\ z = \sum_{i=1}^{m}[\sigma^+(a_i) + \sigma^-(a_i)] \quad (3)$$

subject to

$$\Delta(a_i, a_{i+1}) \geq \delta \text{ if } a_i \succ a_{i+1}\ \forall i \quad (4)$$

$$\Delta(a_i, a_{i+1}) = 0 \text{ if } a_i \sim a_{i+1}\ \forall i \quad (5)$$

$$\sum_{j=1}^{n}\sum_{\ell=1}^{\alpha_{kj}-1} w_{kj\ell} = 1,\ \forall\ k \quad (6)$$

$$w_{kj\ell} \geq 0, \sigma^+(a_i) \geq 0, \sigma^-(a_i) \geq 0,\ \forall\ j, \ell\ k, \text{ and } i \quad (7)$$

To test the existence of multiple or near optimal solutions, we propose the following multi-objective mathematical model:

$$max\ F(x) = (u_{11}(c_{11}), \ldots, u_{1n}(c_{1n}), \ldots, u_{21}(c_{21}), \ldots, u_{2n}(c_{2n}), \ldots, u_{h1}(c_{h1}), \ldots, u_{hn}(c_{hn}))^T \quad (8)$$

on the polyhedron of Constraints (4)-(7) bounded by the new constraint:

$$\sum_{i=1}^{m}[\sigma^+(a_i) + \sigma^-(a_i)] \leq z^* +\ \epsilon, \quad (9)$$

being z* the optimal value of the Model (3)-(7) in Step 4 of the UTASTAR-T algorithm and ϵ a very small positive number.

To solve the multi-objective mathematical model, we used simulation and the weighted sum method, assigning a weight p_j to each $u_{kj}(c_{kj})$ in the multi-objective function (8), and repeating 1.000 times the following steps:
Step 1: Draw random samples for each p_j such as: $p_j \sim U[0, 1], \forall j$;
Step 2: Solve the multi-objective Mathematical Model with the weighted sum method:

$$max\ F(x) = p_1 * (\sum_{k=1}^{h} u_{kj}(c_{k1})) + p_2 * (\sum_{k=1}^{h} u_{kj}(c_{k2})) + \ldots + p_n * (\sum_{k=1}^{h} u_{kn}(c_{kn})), \quad (10)$$

subject to Constraints (4)-(7) and (9);
Step 3: compute the different solution of variables $w_{kj\ell}$.

Next section presents the experiment conducted to validate our proposal.

3 Numerical experiment

We use actual data for ten countries (taken from [1]) to test our approach, according to three criteria, life expectancy at birth (c_1), education (c_2), and gross national income per capita (c_3), over the years 1990, 1995, 2000, 2005, 2010, 2015. We suppose that the decision maker has ranked these ten countries as: Malaysia (MY), Russia (RU), Turkey (TR), Brazil (BR), China (CN), India (IN), Indonesia (ID), Mexico (MX), Philippines (PH), South Africa (ZA); and he/she considered two relevant characteristics to this decision, the average and the trend of the three criteria.

We aim to analyze the DM's preferences based on the given ranking by applying the UTASTAR-T method. Thus, we first carry out a space transformation; we transform the time-space into the characteristic space presented, and then, by applying UTASTAR-T, an optimal solution was reached: objective function (z) = 0; and variables $w_{kj\ell}$ are presented in Table 1. One can see that, in terms of average ($k = 1$), the higher weight is attributed to c_3 (w_{137} = 0.05 and w_{138} = 0.85, amounting to 0.9 for this criterion), whereas in terms of trend ($k = 2$), the higher weight is c_1 (w_{212} = 0.60 and w_{217} = 0.20, amounting to 0.8).

Table 1. $w_{kj\ell}$ weights values obtained by applying UTASTAR-T.

characteristic	$w_{1j\ell}$	Value	characteristic	$w_{2j\ell}$	Value
Average	w_{113}	0.10	Trend	w_{212}	0.60
	w_{137}	0.05		w_{217}	0.20
	w_{138}	0.85		w_{222}	0.05
	-	-		w_{234}	0.15

Table 2. Different $w_{kj\ell}$ weights values obtained by applying the multi-objective model.

w_{111}	w_{117}	w_{126}	w_{128}	w_{138}	w_{139}
0.20 × 7	0.20 × 7	0.2 × 7	0.4 × 7	0.10 × 79	0.00 × 28
0.50 × 3	0.10 × 3	0.1 × 993	0.2 × 3	0.05 × 921	0.05 × 921
0.65 × 18	0.05 × 18	-	0.1 × 18	-	0.10 × 51
0.80 × 972	0.00 × 972	-	0.0 × 972	-	-

Table 2 shows the result by applying simulation and the multi-objective model. Due to space limitation, we only provide the $w_{1j\ell}$ in terms of average (i.e., $k = 1$), and how many times in the simulation each $w_{1j\ell}$ appears. From this table one can observe that most of the time, w_{111} reaches the 0.80 value, which is different from the results in Table 1, where c_3 was the most important criterion for $k = 1$. In the opposite, Table 2 confirms that c_2 is not too relevant in terms of the average since only w_{126} and w_{128} were greater than zero, and their values are not much higher. So, by using simulation and multi-objective model, it is possible to find possible values for the weights.

4 Conclusion

In this paper, we propose an extension of the UTASTAR method to obtain multi-criteria preferences in terms of the criteria average and trend. Our preliminary results show that the analysis of the summary measures can be relevant to certain decisions, and the criteria weights can differ according to the summary measure being considered. Moreover, exploring multiple compatible models using simulation with multi-objective optimization makes it possible to identify some of these models. For future work, we aim to implement another multi-objective method, such as ϵ-constraint.

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